Preliminary Exams: Dr. Kaiser

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# The generalized Gamma Distribution

Suppose that has a standard gamma distribution with the probability density function, for and ,

*Note that this is the alternative form of the gamma distribution P.D.F. and is different than what was given in the problem.* And a generalized gamma distribution results from the transformation

where is a known constant such that . Larger values of result in more extreme right tails in the resulting density function.

## 1.) Derive the P.D.F. of using the Jacobian method

The general formula for the Jacobian transformation is

where the pdf of is given above. And if then . The Jacobian determinant of this term is , and can be substituted for in the distribution function for , our new equation is:

Which, for positive nonzero , , and , simplifies to:

for all values of , 0 elsewhere.

## 2.) Sample Statistics

Simulate a set of data of size from a generalized gamma distribution with , , and . Note that this is most easily accomplished by first simulating the set of data from a gamma distribution with and .

# Simulating Data  
  
## Original Gamma Distribution  
n<-100 # sample size of data  
alpha<-10 # shape parameter  
beta<-3 # scale parameter  
gamma.sim<-data.frame(rgamma(n,shape=alpha,scale=beta))  
colnames(gamma.sim)<-c("Value") # set column name for our 100 simulated values.

### For this example, first use the *original sample of the gamma distribution* to compute:

* The sample mean
* Sample variance
* Skewness

# Sample Mean  
mean(gamma.sim$Value)

## [1] 30.58377

# Variance  
var(gamma.sim$Value)

## [1] 68.97056

# Skewness  
s<-skewness(gamma.sim$Value)  
s[[1]]

## [1] 0.4391151

### Second, use the *generalized gamma function* to compute:

* The sample mean
* Sample variance
* Skewness

# Mew simulated data - Generalized gamma distribution  
f<-function(n,c.gen=3.25,beta.gen=3,alpha.gen=9){  
 (rgamma(n,shape=alpha.gen,scale=beta.gen))^c.gen  
}  
n=100  
gen.gamma.sim<-data.frame(f(n))  
  
# Sample Mean  
mean(gen.gamma.sim$f.n.)

## [1] 66277.92

# Variance  
var(gen.gamma.sim$f.n.)

## [1] 5615913261

# Skewness  
s2<-skewness(gen.gamma.sim$f.n.)  
s2[[1]]

## [1] 2.100211

## 3.) Plot Empirical vs. Theoretical densities of *generalized gamma function*.

Graph the empirical density of the generalized gamma sample (histogram on a probability scale) and overlay a curve giving the theoretical probability density function that was derived in part 1. **The area shaded green is the density curve for the simulated (empirical) data, the red line (without shading) is the density curve for the theoretical density from the formula.**

